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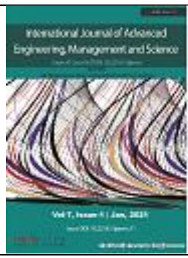

Information Abusing of Rating Agency in "Beauty Contest"

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***Abstract*— This paper studies imitation behavior by subjects in information structure, for example rating agencies. We consider agencies make their rating based on knowledge of prior distribution, public signal, and private signal. They focus on two goals: accurately estimate the risk of the target, avoid giving rate far away from others. We find that agencies will overreact to the prior belief, public signal and underreact to the private signal. And we analyze the welfare the social welfare loss caused by this behavior and the impact on private information acquiring of this behavior.*

***Keywords*— beauty contest, rating agency, signal, information acquaintance, welfare.**

I. INTRODUCTION

Before making an estimation, many subjects seek to not only give an accurate estimation, but also predict other subjects' estimation. Because estimating far away from the majority make himself dubious. And the real value of the objective can only be disclosed after a while or gradually, so being different from others makes his estimation seem inaccurate and impacts his reputation temporarily. Even the real value is disclosed and prove the minority is right finally, the damage has happened. Thus, when making estimation, the subject will not only try to make accurate estimation but also predict what others will estimate.

In beauty contest game, people are asked to choose the most beautiful girl in the list non-publicly. The girl with highest vote will be the winner. And who choose the winner will be rewarded. However, the outcome of the contest does not go along with the designer's mind. Because voters are not only trying to pick up the most beautiful girl but also trying to predict what others will choose in order to be rewarded. The "beauty contest" terminology is drawn from a well-known parable told by Keynes (1936, Chapter 12).Keynes described newspaper-based competitions whose entrants were invited to choose the prettiest faces from a set of photographs, but where it is optimal to nominate the most popular faces but not the prettiest faces.John Duffy and Rosemarie Nagel (1997) study the reference point is not only mean, but also median or maximum. In Potamites and Schotter (2007),they considered the influence of information, all players in a

beauty-contest game receive either public or private advice which is not directly influence the players' payoff. They find that meaningful public advice shifts the observed rationality levels toward higher rationality, indicating that public but not private advice influences the beliefs about other players. The difference in this paper is that the information is stochastic and given objectively. Celen, Kariv, and Schotter (2010) study the impact of naive advice and observational learning on the processing of information in an information cascade experiment. They find that subjects tend to follow the advice of others, but mostly ignore the past behavior of others (even though both types of information are equally informative in their setup).Morris and Shin (2002) say if there is no socially valuable private information, greater provision of public information always increases welfare. Myatt and Wallace (2012) say decision-makers seek actions that are both matched to some unknown underlying feature of the world (a "fundamental" motive) and also matched to the actions taken by others (a "coordination" motive). The participants may welcome any information that helps them to resolve uncertainty about the state of the world and the likely actions of others.

In a political party, members wish to choose the best action of the policy while conforming as closely as possible to the actions of others. Although they would like to make the right policy and make it together, everyone has different opinion on different policies. They learn from the environment to get their own opinion and listen to the leader. Dewan and Myatt (2008) study that in political party leadership is important as public information to guide political members to move. In Dewan and Myatt's model, the leader speaking is likely to public information, and their opinions are their private information. They balance between making the right policy and not being far away from others' proposition. Such games also have been applied to investment games (Angeletos and Pavan, 2004), to monopolistic competition (Hellwig, 2005), to financial markets (Allen, Morris and Shin, 2006), to a range of other economic problems (Angeletos and Pavan, 2007), and many other papers report to variants of the beauty-contest specification.

In reality, there are many phenomena reflecting beauty contest problem. Except policy making, rating agencies also have over-intimidating problem. When rating agency estimate credit risk of government or company, it uses several factors: historical behavior, fundamental analysis, and private investigation. In order to build or keep its reputation among investors, it will try to accurately estimate the risk of a company, a bond, or a stock. However, if it gets some negative or positive signal in private investigation, it may be worry about using this signal in full trust. Because if other agencies do not receive such signal, making absolutely objective based on his own information set may lead to being quietly different from other agencies. The real performance of the bond or stock can only be seen in the future. Nothing can prove whether ratingagency is doing a good job or bad job temporarily. So, if an agency rates some bond or stock quietly different from other agencies, its reputation may be dubious which is bad for it. Therefore, when making rating decision, an agencywill predict what other agencies rate, and give a rate between his objective estimation and others' rating but not the objective estimation.Angeletos Pavan (2007) say that to measure the efficiency of using information, we should compare the equilibrium to the strategy mapping from primitive information to actions that maximizes ex ante utility. As a benchmark, this strategy identifies the best society could do under the sole constraint that information cannot be centralized or otherwise communicated among the players. Comparing equilibrium to this benchmark we use the difference between private and social incentives in the use of available information to measure the information using efficiency.

In our model, we consider agencies use prior distribution information, public information, and private information in estimation. How agencies will react to the signal they get. The efficiency and objectivity of the estimation agencies making. And how the relativity of private information influences the efficiency and objectivity. The target to estimate we name event value to be general.

In section 2, we reintroduce the beauty contest game to describe the imitation phenomenon hindering objective adjustment simply. In section 3, we introduce the model setting. In section 4, we analyze the equilibrium strategies of agencies, the efficiency loss, and the influence of relativity of private information on strategies and efficiency. In section 5, we assume the private information is

endogenous and analyze what factors are in relation with information acquaintance. In section 6, we analyze the factors influencing social welfare loss. In section 7, we give our conclusion.

II. BEAUTY CONTEST MODEL

In a beauty-contest game n players $i = 1, \dots, n$ simultaneously choose a real number $x_i \in X = [0,100]$, x^* is the most beautiful girl in $[0,100]$. The pay-offs depend on the quadratic distance of actions from an unobserved state variable and from the average action, that is $-(x_i - x^*)^2 - (x_i - \bar{x})^2$, for player i, where $\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$. The winner is the participant who gets the highest core. Through the pointing principle, we can see that players have two targets:

1. Accurately figure out the most beautiful girl x^* in the list.
2. Choose the girl close as possible as he can to the average choice of all players.

When n is large enough, if every player knows exactly the most beautiful girl x^*, then the unique Nash Equilibrium is that all players choose x^*. Ifall playersdo not know x^*, then $\forall a \in X$, $x_i = a$ for all i is a Nash Equilibrium. If only one of players assumed to be j knows x^*, the best choice for him is not x^*, but letting $x_j = \frac{x^* + \bar{x}}{2}$. If $\bar{x} \neq x^*$, then the sophisticated player $j$will not choose the most beautiful girl. Because he faces the problem to not only choose the most beautiful girl but also predict what others will choose.

III. THE MODEL

We examine a three-date model that contains continuum rating agencies, indexed by the unit interval $[0,1]$. At date 0, each agency chooses the precision of private signal. At date 1, each agency observes a public signal and his private signal. Agencies choose their estimation based on their public information and private information. At date 2, every agencies' ratingis disclosed and the loss of deviation from average estimation are realized. At date 3, the real rate of target is disclosed and the loss of deviation from real rate of target is realized. We assume the real rate of target is disclosed later than average rating. That is because the real risks of the target take more time to be known by public, so does the real rate of target. It is long enough for public to compare the rating made by all players and utility loss of deviation from average rating to be realized.

3.1 Date 0

Each agency is facing the problem of accurately predicting the real risk of target and being close to other agencies' rating. The two targets can be contradictory to some extends. So, agencies must balance them two. Let the real rate of target be θ.According to the history information, agencies know the ex-ante probability distribution about $\theta$and $\theta$is normally distributed with mean $\bar{\theta}$and precision τ_θ. Let $t_i$be the rating made by agency$i$. And $\bar{t}$is the average rating of all agencies, that is $\bar{t} = \int_0^1 t_i \, di$ .We assume the utility function of agency$i$ is

$$u_i = -c_1(t_i - \theta)^2 - c_2(t_i - \bar{t})^2 \quad (1)$$

, where $i \in [0,1]$. The first term represents utility loss of deviation from real rate. And second term represents utility loss of deviation from average rating. c_1, c_2 represent relative importance between two utility loss described above. The discount effect between date 2 and date 3 is contained in c_1, c_2.

The society wish that the estimation of rating agency as close as possible to the real rate reflecting all kinds of risks of the target. Imitating other rating agency cannot bring benefit to the society. We assume social welfare function is

$$W = -(c_1 + c_2) \int_0^1 (t_i - \theta)^2 di \quad (2)$$

In section 5, we analyze endogenous private information acquaintance. But firstly, we assume that the precision of private information is exogeneous.

3.2 Date 1

Each agency observes a public information S. And agency$i$observes a private information k_i,$i \in [0,1]$, where

$$S = \theta + \varepsilon$$

$$k_i = \theta + \delta_i.$$

ε is normally distributed with mean zero and precision τ_ϵ . δ_i is also normally distributed with mean zero and precision τ_δ . ε and δ_i are pure noise and independent to other random variables. Agency i's information set is $\{S, k_i\}$. Base on public information S and private information k_i agency i choose his rating t_i to maximize his conditional expectation utility $E(u_i|S, k_i)$. We know conditional probability distribution $\theta|_{S,k_i}$ complies with normal distribution $N(\frac{\tau_\theta\bar{\theta}+\tau_\varepsilon S+\tau_\delta k_i}{\tau_\theta+\tau_\varepsilon+\tau_\delta}, \frac{1}{\tau_\theta+\tau_\varepsilon+\tau_\delta})$.

3.3 Date 2

All ratings are disclosed. Agency i's loss of deviation from average rating $-c_2(t_i - \bar{t})^2$ is realized.

3.4 Date 3

The real rate of target is disclosed. Agency i's loss of deviation from real value of event $-c_1(t_i - \theta)^2$ is realized.

$t = 0$	$t = 1$	$t = 2$	$t = 3$
Agencies choose the precision ofprivate information k_i.	Public signal S and pirate signals k_i are realized. Agencies choose their estimations k_i.	$\bar{t}$ are realized. Payoff $-(t_i - \bar{t})^2$ are realized.	θ is realized.Payoffs $-(t_i - \theta)^2$ are realized.

IV. THE ANALYSIS

In this section, we firstly analyze social welfare maximization. Secondly, we study the equilibrium strategy when private information $k_i, i \in [0,1]$ are independent across the Agencies. Then, we assume that the private information is relevant.

4.1 Social welfare maximization

To accurately estimate the rating, the social optimum problem is

$$\max_{\{t_i\}} E(W|S,k) = -(c_1 + c_2)E\left[\int_0^1 (t_i - \theta)^2 di \,|S,k\right], \qquad (3)$$

where $k = \{k_i\}, i \in [0,1]$.

By the property of integration and expectation, the social optimum problem equals to

$$\max_{t_i} E[(t_i - \theta)^2 |S, k_i]. \qquad (4)$$

Therefore, the optimal rating given by rating agency is $t_i^* = \frac{\tau_\theta\bar{\theta}+\tau_\varepsilon S+\tau_\delta k_i}{\tau_\theta+\tau_\varepsilon+\tau_\delta}, i \in [0,1]$, which is exactly the conditional expectation base on public information S and private information k_i. It is exactly the objective estimation of the rate because $t_i^* = E(\theta|S, k_i)$. It is also the most efficient estimation because it maximizes the social welfare.

4.2 Independent private information

When agencies acquire information from different aspects, the randomness can only depend on the way they acquire the information. The noises of private information are i.i.d. At this condition, private information is independent, that is $Cov(k_i, k_j) = 0, \forall i \neq j$.

Proposition 1. When the noises of private signal are independent, there exists a unique equilibrium in which agencies choose estimation

$$t_i = \alpha S + \beta k_i + \gamma \quad (5)$$

with

$$\alpha = \frac{(c_1 + c_2)\tau_\varepsilon}{(c_1 + c_2)(\tau_\theta + \tau_\varepsilon + \tau_\delta) - c_2\tau_\delta},$$

$$\beta = \frac{c_1\tau_\delta}{(c_1 + c_2)(\tau_\theta + \tau_\varepsilon + \tau_\delta) - c_2\tau_\delta},$$

and

$$\gamma = \frac{(c_1 + c_2)\tau_\theta\bar{\theta}}{(c_1 + c_2)(\tau_\theta + \tau_\varepsilon + \tau_\delta) - c_2\tau_\delta}.$$

Note that $\alpha + \beta + \frac{\gamma}{\bar{\theta}} = 1$.

Proposition 1 characterized how agencies give their rating using information. The strategy they give is different from efficient strategy. This is because when agencies give their rating of the target, they estimate both real rate of the target and other players' rate of target. So, compare to the objective rating, ex-ante information about the rate distribution and public signalare more important, private signal is less important. In proposition 2, we describe this wrong information using.

Proposition 2. In order to maximize agencies' own utility, agencies will overreact to the prior distribution of the real rate and public signal, but underreact to the private signal.

Proof. We have stated that each agency's efficient rate should equal to conditional expectation of the event value θ given his information set, that is t^*. But they act by strategy proved in proposition 1, that is

$$t_i = \alpha S + \beta k_i + \gamma.$$

$$\alpha = \frac{(c_1 + c_2)\tau_\varepsilon}{(c_1 + c_2)(\tau_\theta + \tau_\varepsilon + \tau_\delta) - c_2\tau_\delta} = \frac{\tau_\varepsilon}{\tau_\theta + \tau_\varepsilon + \tau_\delta - \frac{c_2}{c_1+c_2}\tau_\delta} > \frac{\tau_\varepsilon}{\tau_\theta + \tau_\varepsilon + \tau_\delta},$$

$$\beta = \frac{c_1\tau_\delta}{(c_1 + c_2)(\tau_\theta + \tau_\varepsilon + \tau_\delta) - c_2\tau_\delta} = \frac{\tau_\delta}{\tau_\theta + \tau_\varepsilon + \tau_\delta + \frac{c_2}{c_1}(\tau_\theta + \tau_\varepsilon)} < \frac{\tau_\delta}{\tau_\theta + \tau_\varepsilon + \tau_\delta},$$

$$\frac{\gamma}{\bar{\theta}} = \frac{(c_1 + c_2)\tau_\theta}{(c_1 + c_2)(\tau_\theta + \tau_\varepsilon + \tau_\delta) - c_2\tau_\delta} = \frac{\tau_\theta}{\tau_\theta + \tau_\varepsilon + \tau_\delta - \frac{c_2}{c_1+c_2}\tau_\delta} > \frac{\tau_\theta}{\tau_\theta + \tau_\varepsilon + \tau_\delta}.$$

Thus, we have proved that agency overreact to the prior distribution of the real rate and public signal, but underreact to the private signal.

Proposition 2 says, when ratingagencies put more weight on prior distribution of real rate and public signal comparing to the conditional expectation given his information set. At the meantime, agencies put less weight on their private signals comparing to the conditional expectation given his information set accordingly. It is because that when make estimations, agencies not only consider to accurately estimate the real rate of target, but also try to not deviate from average rating made by all agencies. Prior distribution and public signal are known to all players, they are beneficial to accurately estimate both event value and other agencies' estimation. However, even private signal gives some information of predicting the real rate of target, only if it is not certain, it may lead agency's estimation far away from other agencies' rating. We analyze the influence of private information precision on strategy later. Thus, prior distribution and public signal are more important for players than private signal.

4.2 Relevant private information

When agencies acquire information in a similar way, there may be relativity in the noises of information they acquiring. At this condition, private signalsis relevant. We assume that the covariances between noises of private signals are same, that is $Cov(\delta_i, \delta_j) = \frac{\xi}{\tau_\delta}, \forall i \neq j$.

Proposition 3. When the noises of private signals are

relevant, there exists a unique equilibrium in which agencies choose to rate

$$\tilde{t}_i = \tilde{\alpha}S + \tilde{\beta}k_i + \tilde{\gamma} \quad (6)$$

with

$$\tilde{\alpha} = \frac{(c_1+c_2)\tau_\varepsilon - c_2\tau_\varepsilon\xi}{(c_1+c_2)(\tau_\theta+\tau_\varepsilon+\tau_\delta) - c_2((\tau_\theta+\tau_\varepsilon)\xi+\tau_\delta)},$$

$$\tilde{\beta} = \frac{c_1\tau_\delta}{(c_1+c_2)(\tau_\theta+\tau_\varepsilon+\tau_\delta) - c_2((\tau_\theta+\tau_\varepsilon)\xi+\tau_\delta)},$$

and

$$\tilde{\gamma} = \frac{(c_1+c_2)\tau_\theta\bar{\theta} - c_2\tau_\theta\bar{\theta}\xi}{(c_1+c_2)(\tau_\theta+\tau_\varepsilon+\tau_\delta) - c_2((\tau_\theta+\tau_\varepsilon)\xi+\tau_\delta)}$$

Note that $\tilde{\alpha}+\tilde{\beta}+\frac{\tilde{\gamma}}{\bar{\theta}} = 1$.

We can see when $\xi = 0$, the strategy in proposition 2 is as same as in proposition 1, noises of private signals are independent. When $\xi = 1$, private signalsare completely relevant, that means all private information become public information. The weights agency place on signal S, signalsk_i, and prior distribution of event value are $\tilde{\alpha} = \frac{\tau_\varepsilon}{\tau_\theta+\tau_\varepsilon+\tau_\delta}$, $\tilde{\beta} = \frac{\tau_\delta}{\tau_\theta+\tau_\varepsilon+\tau_\delta}$, $\tilde{\gamma} = \frac{\tau_\theta}{\tau_\theta+\tau_\varepsilon+\tau_\delta}$, which is exactly objective efficient estimation that should be made based on information$S$,k_i, and prior distribution of event value. So next proposition we analyze how relativity of private signals affect the objectivity and efficiency of rating made by players.

Proposition 4. More relativity between private noises, more efficient the rating is by players' optimal strategies.

Proof. Take derivative of $\tilde{\alpha}$,$\tilde{\beta}$, and$\frac{\tilde{\gamma}}{\bar{\theta}}$with respect to ξrespectively, we have

$$\frac{\partial\tilde{\alpha}}{\partial\xi} = -\frac{c_1c_2\tau_\varepsilon\tau_\delta}{[(c_1+c_2)(\tau_\theta+\tau_\varepsilon+\tau_\delta) - c_2(\tau_\delta+(\tau_\theta+\tau_\varepsilon)\xi)]^2} < 0$$

$$\frac{\partial\tilde{\beta}}{\partial\xi} = \frac{c_2(\tau_\theta+\tau_\varepsilon)}{[(c_1+c_2)(\tau_\theta+\tau_\varepsilon+\tau_\delta) - c_2(\tau_\delta+(\tau_\theta+\tau_\varepsilon)\xi)]^2} > 0$$

$$\frac{\partial(\frac{\tilde{\gamma}}{\bar{\theta}})}{\partial\xi} = -\frac{c_1c_2\tau_\theta\tau_\delta}{[(c_1+c_2)(\tau_\theta+\tau_\varepsilon+\tau_\delta) - c_2(\tau_\delta+(\tau_\theta+\tau_\varepsilon)\xi)]^2} < 0.$$

In proposition 2, we have proved that $\alpha > \frac{\tau_\varepsilon}{\tau_\theta+\tau_\varepsilon+\tau_\delta}$,$\beta < \frac{\tau_\delta}{\tau_\theta+\tau_\varepsilon+\tau_\delta}$, and $\frac{\gamma}{\bar{\theta}} > \frac{\tau_\theta}{\tau_\theta+\tau_\varepsilon+\tau_\delta}$, which are $\tilde{\alpha}$,$\tilde{\beta}$, and $\frac{\tilde{\gamma}}{\bar{\theta}}$with $\xi = 0$.Thus as ξincreases from 0 to 1, $\tilde{\alpha}$, $\tilde{\beta}$, and $\frac{\tilde{\gamma}}{\bar{\theta}}$are getting close to $\frac{\tau_\varepsilon}{\tau_\theta+\tau_\varepsilon+\tau_\delta}$,$\frac{\tau_\delta}{\tau_\theta+\tau_\varepsilon+\tau_\delta}$, and $\frac{\tau_\theta}{\tau_\theta+\tau_\varepsilon+\tau_\delta}$. So, the rating is more efficient as ξincreases.

□

As the relativity of private noises increases, the private signals become more relevant. The signals agencies get from private signal is more similar. Thus, agency can make their rating base more on private signals because they worry less about being different from other agencies. In other words, as the relativity of private noises increase, the private information becomes more public. Using this information to rate will cause less balance problem between accurately rate the target and closing to average rating.

V. WELFARE ANALYSIS

In this section, we consider social welfare loss. Substitute efficient strategy and strategy proved by proposition 1 into unconditional expected social welfare function respectively, we have

$$E(W^*) = -(c_1+c_2)E\int_0^1\left(\frac{\tau_\theta\bar{\theta}+\tau_\varepsilon S+\tau_\delta k_i}{\tau_\theta+\tau_\varepsilon+\tau_\delta} - \theta\right)^2 di$$

$$= -(c_1+c_2)E\left(\frac{\tau_\theta\bar{\theta}+\tau_\varepsilon S+\tau_\delta k_i}{\tau_\theta+\tau_\varepsilon+\tau_\delta} - \theta\right)^2$$

$$= -\frac{c_1 + c_2}{\tau_\theta + \tau_\varepsilon + \tau_\delta} \tag{7}$$

$$E(W) = -(c_1 + c_2)E\int_0^1 (\alpha S + \beta k_i + \gamma - \theta)^2 di$$

$$= -(c_1 + c_2)E(\alpha S + \beta k_i + \gamma - \theta)^2$$

$$= -(c_1 + c_2)\frac{(c_1 + c_2)^2(\tau_\theta + \tau_\varepsilon + \tau_\delta) - (2c_1c_2 + c_2^2)\tau_\delta}{[(c_1 + c_2)(\tau_\theta + \tau_\varepsilon + \tau_\delta) - c_2\tau_\delta]^2} \tag{8}$$

Divide (8) by(7), we get

$$\frac{E(W^*)}{E(W)} = \frac{1}{\tau_\theta + \tau_\varepsilon + \tau_\delta}\frac{[(c_1 + c_2)(\tau_\theta + \tau_\varepsilon + \tau_\delta) - c_2\tau_\delta]^2}{(c_1 + c_2)^2(\tau_\theta + \tau_\varepsilon + \tau_\delta) - (2c_1c_2 + c_2^2)\tau_\delta}$$

$$= \frac{(c_1 + c_2)^2(\tau_\theta + \tau_\varepsilon + \tau_\delta)^2 - 2(c_1 + c_2)c_2(\tau_\theta + \tau_\varepsilon + \tau_\delta)\tau_\delta + c_2^2\tau_\delta^2}{(c_1 + c_2)^2(\tau_\theta + \tau_\varepsilon + \tau_\delta)^2 - 2(c_1 + c_2)c_2(\tau_\theta + \tau_\varepsilon + \tau_\delta)\tau_\delta + c_2^2\tau_\delta^2 + c_2^2\tau_\delta(\tau_\theta + \tau_\varepsilon)}$$

$$= \frac{1}{1 + \frac{c_2^2\tau_\delta(\tau_\theta + \tau_\varepsilon)}{[(c_1 + c_2)(\tau_\theta + \tau_\varepsilon + \tau_\delta) - c_2\tau_\delta]^2}} < 1$$

Without loss of generality, we normalize $c_1 + c_2 = 1, c_1, c_2 > 0$, and define welfare loss rate function as

$$f(c_2, \tau_\theta, \tau_\varepsilon, \tau_\delta) = \frac{c_2^2\tau_\delta(\tau_\theta + \tau_\varepsilon)}{[\tau_\theta + \tau_\varepsilon + \tau_\delta - c_2\tau_\delta]^2}. \tag{9}$$

The larger f is, the more proportion of social welfare is lost. In order to analyze the influence of $c_2, \tau_\theta, \tau_\varepsilon, \tau_\delta$ on social welfare loss rate. We take derivative of f with respect to $c_2, \tau_\theta, \tau_\varepsilon, \tau_\delta$ respectively. Clearly

$$\frac{\partial f}{\partial c_2} > 0,$$

that is the more agencies care about rating close to average rating, the more social welfare loss. It is intuitive, if agency cares more about their temporary reputation rather than estimate the risk of target the social welfare will decrease.

$$\frac{\partial f}{\partial \tau_\theta} = \frac{c_2^2\tau_\delta[(1 - c_2)\tau_\delta - \tau_\theta - \tau_\varepsilon]}{[\tau_\theta + \tau_\varepsilon + \tau_\delta - c_2\tau_\delta]^3} = 0$$

We have $\tau_\theta = \max\{0, (1 - c_2)\tau_\delta - \tau_\varepsilon\}$, if $(1 - c_2)\tau_\delta - \tau_\varepsilon < 0, \frac{\partial f}{\partial \tau_\theta} < 0$ for $\tau_\theta > 0$. The social welfare loss rate decreases with precision of prior distribution. If $(1 - c_2)\tau_\delta - \tau_\varepsilon > 0$, the loss proportion of social welfare increases on $\tau_\theta \in [0, (1 - c_2)\tau_\delta - \tau_\varepsilon]$ and decreases on $[(1 - c_2)\tau_\delta - \tau_\varepsilon, \infty)$.As $\tau_\theta \to \infty, f \to 0$. When the precision of public signal is enough higher than private signal, the loss proportion decreases as τ_θ increases. Otherwise, the loss proportion increases first and then decreases. When prior distribution is perfectly accurate, it is best for agencies to focus on unconditional expectation. It is because when the precision of prior distribution is low, the relative importance of private signal is large. This leads to a increasing proportion loss of social welfare. When the precision of prior distribution is high, agencies will focus more on this prior information and the relative importance of private signal is small. This leads the strategy of agencies being closer to social optimum strategy. Thus, the loss proportion decreases as τ_θ increases.

For $\frac{\partial f}{\partial \tau_\varepsilon} = \frac{\partial f}{\partial \tau_\theta}$, the analysis of the precision of public signal is similar. And

$$\frac{\partial f}{\partial \tau_\delta} = \frac{c_2^2(\tau_\theta + \tau_\varepsilon)(\tau_\theta + \tau_\varepsilon - (1-c_2)\tau_\delta)}{[\tau_\theta + \tau_\varepsilon + \tau_\delta - c_2\tau_\delta]^3} = 0.$$

We have $\tau_\delta = \frac{\tau_\theta+\tau_\varepsilon}{1-c_2} > 0$. If $\tau_\delta \in [0, \frac{\tau_\theta+\tau_\varepsilon}{1-c_2}]$, $\frac{\partial f}{\partial \tau_\delta} > 0$, the loss proportion increase as τ_δ increases. If $\tau_\delta \in [\frac{\tau_\theta+\tau_\varepsilon}{1-c_2}, \infty)$, $\frac{\partial f}{\partial \tau_\delta} < 0$, the loss proportion decreases as τ_δ increases. When τ_δ is low, agencies are cautious to use private signals. This leads to more difference between agency's strategy and efficient strategy. Thus, the loss proportion increases. When τ_δ is high, agencies can trust their private signals because others also rely on private signal more than before. It helps agency to rate accurately and be close to other's rating.

VI. ENDOGENOUS INFORMATION ACQUAINTANCE

In this section, we assume the private signal noises are independent for simplicity. We first analyze the impact of precision of private information on how agencies use prior information, public signals, and private signals, and the impact on the efficiency of rating. Then we assume that agencies can increase the precision of private signal at some cost and do some comparative static analysis.

In proposition 1, we prove that the optimal estimation strategy for player is

$$t_i = \alpha S + \beta k_i + \gamma$$

with

$$\alpha = \frac{(c_1+c_2)\tau_\varepsilon}{(c_1+c_2)(\tau_\theta+\tau_\varepsilon+\tau_\delta) - c_2\tau_\delta},$$

$$\beta = \frac{c_1\tau_\delta}{(c_1+c_2)(\tau_\theta+\tau_\varepsilon+\tau_\delta) - c_2\tau_\delta},$$

and

$$\gamma = \frac{(c_1+c_2)\tau_\theta\bar{\theta}}{(c_1+c_2)(\tau_\theta+\tau_\varepsilon+\tau_\delta) - c_2\tau_\delta}.$$

Take derivative of $\alpha - \frac{\tau_\varepsilon}{\tau_\theta+\tau_\varepsilon+\tau_\delta}$ with respect to τ_δ,

$$\frac{\partial(\alpha - \frac{\tau_\varepsilon}{\tau_\theta+\tau_\varepsilon+\tau_\delta})}{\partial \tau_\delta} = \frac{c_2(c_1+c_2)(\tau_\theta+\tau_\varepsilon)^2 - c_1c_2\tau_\delta^2}{[(c_1+c_2)(\tau_\theta+\tau_\varepsilon+\tau_\delta) - c_2\tau_\delta]^2(\tau_\theta+\tau_\varepsilon+\tau_\delta)^2}\tau_\varepsilon = 0,$$

we have $\tau_\delta = \sqrt{\frac{c_1+c_2}{c_1}}(\tau_\theta+\tau_\varepsilon)$. When $\tau_\delta < \sqrt{\frac{c_1+c_2}{c_1}}(\tau_\theta+\tau_\varepsilon)$, $\frac{\partial(\alpha - \frac{\tau_\varepsilon}{\tau_\theta+\tau_\varepsilon+\tau_\delta})}{\partial \tau_\delta} > 0$. When $\tau_\delta > \sqrt{\frac{c_1+c_2}{c_1}}(\tau_\theta+\tau_\varepsilon)$, $\frac{\partial(\alpha - \frac{\tau_\varepsilon}{\tau_\theta+\tau_\varepsilon+\tau_\delta})}{\partial \tau_\delta} < 0$.The difference of weight put on public signal firstly decreases and then increases. Because as precision of private signal τ_δ increases, both optimal strategy and efficient strategy put less weight on public information. The speed of weight decreasing of optimal strategy is slower than it of efficient strategy when τ_δ is small. As τ_δ exceeds $\sqrt{\frac{c_1+c_2}{c_1}}(\tau_\theta+\tau_\varepsilon)$, the speed of weight decreasing of optimal strategy is faster than it of efficient strategy. As τ_δ becomes infinite, the optimal strategy becomes also efficient. That is

$$\lim_{\tau_\delta \to \infty} \alpha - \frac{\tau_\varepsilon}{\tau_\theta + \tau_\varepsilon + \tau_\delta} = 0.$$

Take derivative of $\beta - \frac{\tau_\delta}{\tau_\theta+\tau_\varepsilon+\tau_\delta}$with respective to τ_δ,

$$\frac{\partial(\beta - \frac{\tau_\delta}{\tau_\theta+\tau_\varepsilon+\tau_\delta})}{\partial \tau_\delta} = \frac{c_1 c_2 \tau_\delta^2 - c_2(c_1+c_2)(\tau_\theta+\tau_\varepsilon)^2}{[(c_1+c_2)(\tau_\theta+\tau_\varepsilon+\tau_\delta) - c_2\tau_\delta]^2(\tau_\theta+\tau_\varepsilon+\tau_\delta)^2}(\tau_\theta+\tau_\varepsilon) = 0,$$

we have $\tau_\delta = \sqrt{\frac{c_1+c_2}{c_1}}(\tau_\theta+\tau_\varepsilon)$ .When $\tau_\delta < \sqrt{\frac{c_1+c_2}{c_1}}(\tau_\theta+\tau_\varepsilon)$, $\frac{\partial(\beta - \frac{\tau_\delta}{\tau_\theta+\tau_\varepsilon+\tau_\delta})}{\partial \tau_\delta} < 0$. When $\tau_\delta > \sqrt{\frac{c_1+c_2}{c_1}}(\tau_\theta+\tau_\varepsilon)$, $\frac{\partial(\beta - \frac{\tau_\delta}{\tau_\theta+\tau_\varepsilon+\tau_\delta})}{\partial \tau_\delta} > 0$.The difference of weight put on private signals firstly decreases and then increases. Because as precision of private information $\tau_\delta$increases, both optimal strategy and efficient strategy put more weight on public information. The speed of weight increasing of optimal strategy is slower than it of efficient strategy when $\tau_\delta$is small. As $\tau_\delta$exceeds $\sqrt{\frac{c_1+c_2}{c_1}}(\tau_\theta+\tau_\varepsilon)$, the speed of weight increasing of optimal strategy is faster than it of efficient strategy. As $\tau_\delta$becomes infinite, the optimal strategy becomes also efficient. That is

$$\lim_{\tau_\delta \to \infty} \beta - \frac{\tau_\delta}{\tau_\theta + \tau_\varepsilon + \tau_\delta} = 0.$$

Take derivative of $\frac{\gamma}{\bar{\theta}} - \frac{\tau_\theta}{\tau_\theta+\tau_\varepsilon+\tau_\delta}$with respective to τ_δ,

$$\frac{\partial(\frac{\gamma}{\bar{\theta}} - \frac{\tau_\theta}{\tau_\theta+\tau_\varepsilon+\tau_\delta})}{\partial \tau_\delta} = \frac{c_2(c_1+c_2)(\tau_\theta+\tau_\varepsilon)^2 - c_1 c_2 \tau_\delta^2}{[(c_1+c_2)(\tau_\theta+\tau_\varepsilon+\tau_\delta) - c_2\tau_\delta]^2(\tau_\theta+\tau_\varepsilon+\tau_\delta)^2}\tau_\theta = 0,$$

we have $\tau_\delta = \sqrt{\frac{c_1+c_2}{c_1}}(\tau_\theta+\tau_\varepsilon)$.The difference of weight put on prior distribution firstly decreases and then increases. Because as precision of private information $\tau_\delta$increases, both optimal strategy and efficient strategy put less weight on prior expectation. The speed of weight decreasing of optimal strategy is slower than it of efficient strategy when $\tau_\delta$is small. As $\tau_\delta$exceeds $\sqrt{\frac{c_1+c_2}{c_1}}(\tau_\theta+\tau_\varepsilon)$, the speed of weight decreasing of optimal strategy is faster than it of efficient strategy. As $\tau_\delta$becomes infinite, the optimal strategy becomes also efficient. That is

$$\lim_{\tau_\delta \to \infty} \frac{\gamma}{\bar{\theta}} - \frac{\tau_\theta}{\tau_\theta + \tau_\varepsilon + \tau_\delta} = 0.$$

The differences of weight put on prior distribution, public information, and private information all decrease on $[0, \sqrt{\frac{c_1+c_2}{c_1}}(\tau_\theta+\tau_\varepsilon)]$and increase on $\left[\sqrt{\frac{c_1+c_2}{c_1}}(\tau_\theta+\tau_\varepsilon), +\infty\right)$.The difference between two strategies become maximal when $\tau_\delta = \sqrt{\frac{c_1+c_2}{c_1}}(\tau_\theta+\tau_\varepsilon)$.The optimal strategy $t_{\tau_\delta}(S, k_i)$converges to efficient strategy t^*as $\tau_\delta$becomes infinite.

From the analysis above, we know that as each agency's private information becomes more accurate, the rating they give will be closer to efficient strategy if $\tau_\delta$is sufficiently large. Next, we want to find out if only one agency's private information becomes more accurate and other players' precision of private information is unchanged, will this agency give more efficient estimation? Even when his private information becomes perfectly accurate, will he against the pressure of peers and make objective and efficient estimation?

Proposition 5. Given other agencies' precision of private signal τ_δ same and unchanged, agency i's rating will be more efficient as his precision of private signal τ_{δ_i} increases. Specially, when agency i's private signal is perfectly accurate, he will give efficient rating, that is $t^* = k_i$.

Proof. Given all other agencies' precision of private signals is τ_δ, $\bar{t}|_{S,k_i} \sim N(\alpha S + \beta \frac{\tau_\theta \bar{\theta} + \tau_\varepsilon S + \tau_{\delta_i} k_i}{\tau_\theta + \tau_\varepsilon + \tau_{\delta_i}} + \gamma, \frac{1}{\tau_\theta + \tau_\varepsilon + \tau_{\delta_i}} + \frac{1}{\tau_\delta})$. The optimal problem in section 3 becomes

$$\max -c_1 \left[t_i^2 - 2\frac{\tau_\theta \bar{\theta} + \tau_\varepsilon S + \tau_{\delta_i} k_i}{\tau_\theta + \tau_\varepsilon + \tau_{\delta_i}} t_i + \frac{1}{\tau_\theta + \tau_\varepsilon + \tau_{\delta_i}} + \left(\frac{\tau_\theta \bar{\theta} + \tau_\varepsilon S + \tau_{\delta_i} k_i}{\tau_\theta + \tau_\varepsilon + \tau_{\delta_i}} \right)^2 \right]$$

$$-c_2 \left[\beta^2 \left(\frac{1}{\tau_\theta + \tau_\varepsilon + \tau_{\delta_i}} + \frac{1}{\tau_\delta} \right) + \left(t_i - \alpha S - \beta \frac{\tau_\theta \bar{\theta} + \tau_\varepsilon S + \tau_{\delta_i} k_i}{\tau_\theta + \tau_\varepsilon + \tau_{\delta_i}} - \gamma \right)^2 \right].$$

The first order condition with respect to t_i is same to section 3. Thus the optimal rating for agency i is still $t_i = \alpha S + \beta k_i + \gamma$, where $\alpha = \frac{(c_1+c_2)\tau_\varepsilon}{(c_1+c_2)(\tau_\theta+\tau_\varepsilon+\tau_{\delta_i}) - c_2\tau_{\delta_i}}$, $\beta = \frac{c_1\tau_\delta}{(c_1+c_2)(\tau_\theta+\tau_\varepsilon+\tau_{\delta_i}) - c_2\tau_{\delta_i}}$, and $\gamma = \frac{(c_1+c_2)\tau_\theta\bar{\theta}}{(c_1+c_2)(\tau_\theta+\tau_\varepsilon+\tau_{\delta_i}) - c_2\tau_{\delta_i}}$.

Take limitation of $\alpha, \beta, \frac{\gamma}{\bar{\theta}}$, we have $\lim_{\tau_i \to \infty} \alpha = 0$, $\lim_{\tau_i \to \infty} \beta = 1$, and $\lim_{\tau_i \to \infty} \frac{\gamma}{\bar{\theta}} = 0$.□

From proposition 5, we know that when agency has absolute confidence in his private information, no matter what optimal estimation other agencies will give for any precision of others' private signals, this agency will give his real rate estimation of target. It is like keeping the truth in minority.

In the next, we assume agencies can do some research at some cost to increase the precision of his private signal. The cost of research is

$$c(\tau_\delta) = \frac{1}{2}\lambda\tau_\delta^2.$$

At the date 0 before get public signal and private signal, agencies must choose the precision of his private signal. Thus, the unconditional expectation utility maximizing problem is

$$\max Eu = -c_1[Var(\alpha S + \beta k_i + \gamma - \theta) + [(\alpha + \beta - 1)\bar{\theta} + \gamma]^2] - c_2\beta^2 Var(\delta_i) - \frac{1}{2}\lambda\tau_\delta^2$$

Simplify first order condition with respect to τ_δ, we have

$$c_1 + c_2 = \lambda\tau_\delta \left[\tau_\theta + \tau_\varepsilon + \tau_\delta + \frac{c_2}{c_1}(\tau_\theta + \tau_\varepsilon) \right]^2. \qquad (*)$$

The proof of first order condition is presented in appendix.

Through $(*)$ we can easily see that the precision of prior distribution and public information are substitutes of private information. If the precisions of prior distribution and public signal decrease, player will tend to do more research to increase the precision of private signal. And if the cost of research λ increases, agencies will do less research which is compatible to our intuitive.

Proposition 6. If the agency cares less about estimating closely to other agencies' and cares more about rating the risk of target accurately, he will expense more to do research.

Proof.c_1,$c_2$represent the relative importance between accurately rating the target and estimating closely to other agencies' estimation. Thus, we can normalize c_1,$c_2$by letting $c_1 + c_2 = 1$, then we have

$$\lambda\tau_\delta\left[\tau_\theta + \tau_\varepsilon + \tau_\delta + \frac{1-c_1}{c_1}(\tau_\theta + \tau_\varepsilon)\right]^2 = 1$$

It is obvious that $\tau_\delta$increases as $c_1$increases. □

VII. SUMMARY

In this paper, we examine beauty contest model with information structure to describe a phenomenon that in order to keep reputation rating agency are going to miss use the information they get to rate. We consider three kinds of information: prior distribution, public information, and private information. And prove the unique equilibrium strategy of agency. We find that agencies use the signals in a way that is not efficient and objective.

The prior distribution and public signal have two parts of effect: first, it informs agency of the real risk rate of the target; second, it informs the agency of the likely action of other agencies. However, the private signal only has the first effect. Thus, agency overreacts to the prior distribution and public information but underreacts to private information which have same valuable information. This kind of imitation leads to inaccurate estimation. To be further, we consider what will happen if private signals of agencies are relative. When private signals are relative, private signal has two parts of effect too like prior distribution and public information. We find that more relative private signals are, the estimation they give will be closer to efficiency and objectivity. That is because as the relativity of private information increases the private information of agencies become more "public", and the second effect increases. When an agency gets a private signal, other agencies are likely to get same private signal. If private signals become perfectly relative, the private signals become totally public, then the estimation is efficient and objective. In social welfare analysis, we find that the proportion of social welfare loss decreases as the precision of prior distribution and public signal increase if agencies care more about deviation from average rating. The proportion of social welfare loss decreases as the precisions of prior distribution and public signal are sufficiently large respectively. Andthe proportion of social welfare loss increases firstly and then decrease as the precision of private signals increases. Lastly, we analyze endogenous information acquaintance. As the precision of private information increases, the efficiency of estimation decreases at first and then increases. Keep precision of private signals of other agencies unchanged, if just one agency's precision of private signal increase he will make more efficient and objective estimation. Specially, when the private signal of the agency is perfectly accurate, that is the agency has full knowledge of the event value, he will make efficient and objective rating estimation no matter what others do. It explains the phenomenon that the truth is generally held by the minority. In analyze what research agencies will do to increase the precision of private signal, we find that when precision of prior distribution and public signal is high agencies do less research to increase precision of private information. When agencies care more about the loss of deviation from others than the loss of deviation from the truth, they do less research.

Appendix

The proof of proposition 1.

Suppose that each agency forms a linear conjecture on equilibrium estimation

$$t_i = \alpha S + \beta k_i + \gamma.$$

According to the law of large numbers, the average estimation become

$$\bar{t} = \alpha S + \beta \theta + \gamma.$$

$\bar{t}|_{S,k_i}$ complies with normal distribution $N(\alpha S + \beta\bar{\theta} + \gamma, \frac{\beta^2}{\tau_\theta+\tau_\varepsilon+\tau_\delta})$.

Conditional expectations utility of agency i are

$E(u_i|S,k_i) = E[-c_1(t_i-\theta)^2 - c_2(t_i-\bar{t})^2|S,k_i]$

$$= -c_1\left(t_i^2 - 2t_i\frac{\tau_\theta\bar{\theta}+\tau_\varepsilon S+\tau_\delta k_i}{\tau_\theta+\tau_\varepsilon+\tau_\delta} + \frac{1}{\tau_\theta+\tau_\varepsilon+\tau_\delta} + \left(\frac{\tau_\theta\bar{\theta}+\tau_\varepsilon S+\tau_\delta k_i}{\tau_\theta+\tau_\varepsilon+\tau_\delta}\right)^2\right)$$

$$-c_2\left(\frac{\beta^2}{\tau_\theta+\tau_\varepsilon+\tau_\delta} + \left(t_i - \alpha S - \beta\frac{\tau_\theta\bar{\theta}+\tau_\varepsilon S+\tau_\delta k_i}{\tau_\theta+\tau_\varepsilon+\tau_\delta} - \gamma\right)^2\right)$$

To maximize $E(u_i|S,k_i)$, we get first order condition with respect to t_i

$$c_1\frac{\tau_\theta\bar{\theta}+\tau_\varepsilon S+\tau_\delta k_i}{\tau_\theta+\tau_\varepsilon+\tau_\delta} + c_2\left(\alpha S + \beta\frac{\tau_\theta\bar{\theta}+\tau_\varepsilon S+\tau_\delta k_i}{\tau_\theta+\tau_\varepsilon+\tau_\delta} + \gamma\right) = (c_1+c_2)(\alpha S + \beta k_i + \gamma).$$

This equation satisfies with all S and k_i. Thus, we get

$$c_1\frac{\tau_\varepsilon}{\tau_\theta+\tau_\varepsilon+\tau_\delta} + c_2\alpha + c_2\frac{\beta\tau_\varepsilon}{\tau_\theta+\tau_\varepsilon+\tau_\delta} = (c_1+c_2)\alpha,$$

$$c_1\frac{\tau_\delta}{\tau_\theta+\tau_\varepsilon+\tau_\delta} + c_2\frac{\beta\tau_\delta}{\tau_\theta+\tau_\varepsilon+\tau_\delta} = (c_1+c_2)\beta,$$

and

$$c_1\frac{\tau_\theta\bar{\theta}}{\tau_\theta+\tau_\varepsilon+\tau_\delta} + c_2\left(\frac{\beta\tau_\theta\bar{\theta}}{\tau_\theta+\tau_\varepsilon+\tau_\delta} + \gamma\right) = (c_1+c_2)\gamma.$$

Solve the equations, we get

$$\alpha = \frac{(c_1+c_2)\tau_\varepsilon}{(c_1+c_2)(\tau_\theta+\tau_\varepsilon+\tau_\delta) - c_2\tau_\delta},$$

$$\beta = \frac{c_1\tau_\delta}{(c_1+c_2)(\tau_\theta+\tau_\varepsilon+\tau_\delta) - c_2\tau_\delta},$$

$$\gamma = \frac{(c_1+c_2)\tau_\theta\bar{\theta}}{(c_1+c_2)(\tau_\theta+\tau_\varepsilon+\tau_\delta) - c_2\tau_\delta}. \quad \square$$

The proof of proposition 3.

Suppose that each agency forms a linear conjecture on equilibrium estimation

$$\tilde{t}_i = \tilde{\alpha}S + \tilde{\beta}k_i + \tilde{\gamma}.$$

The average estimation becomes

$$\bar{t} = \tilde{\alpha}S + \tilde{\beta}\int_0^1 k_i\, di + \tilde{\gamma}.$$

According to the law of large numbers, we have

$$E\left(\int_0^1 k_i\, di \middle| S, k_i\right) = E(\theta|S,k_i) + E(\delta_j|S,k_i) = \frac{\tau_\theta\bar{\theta} + \tau_\varepsilon S + \tau_\delta k_i}{\tau_\theta + \tau_\varepsilon + \tau_\delta} + \frac{(\tau_\theta + \tau_\varepsilon)k_i - \tau_\theta\bar{\theta} - \tau_\varepsilon S}{\tau_\theta + \tau_\varepsilon + \tau_\delta}\xi$$

$$Var\left(\int_0^1 k_i\, di \middle| S, k_i\right) = \lim_{n\to\infty} Var\left(\frac{\sum_{m=1}^n k_m}{n}\right) = \frac{\xi}{\tau_\delta}$$

Thus, $\bar{t}|_{S,k_i}$ complies with normal distribution $N(\tilde{\alpha}S + \tilde{\beta}(\frac{\tau_\theta\bar{\theta}+\tau_\varepsilon S+\tau_\delta k_i}{\tau_\theta+\tau_\varepsilon+\tau_\delta} + \frac{(\tau_\theta+\tau_\varepsilon)k_i-\tau_\theta\bar{\theta}-\tau_\varepsilon S}{\tau_\theta+\tau_\varepsilon+\tau_\delta}\xi) + \tilde{\gamma}, \frac{\tilde{\beta}^2\xi}{\tau_\delta})$.

Conditional expectations utility of agency$i$are

$E(u_i|S,k_i) = E[-c_1(\tilde{t}_i - \theta)^2 - c_2(\tilde{t}_i - \bar{t})^2|S,k_i]$

$$= -c_1\left(\tilde{t}_i^{\,2} - 2\tilde{t}_i\frac{\tau_\theta\bar{\theta} + \tau_\varepsilon S + \tau_\delta k_i}{\tau_\theta + \tau_\varepsilon + \tau_\delta} + \frac{1}{\tau_\theta + \tau_\varepsilon + \tau_\delta} + \left(\frac{\tau_\theta\bar{\theta} + \tau_\varepsilon S + \tau_\delta k_i}{\tau_\theta + \tau_\varepsilon + \tau_\delta}\right)^2\right)$$

$$-c_2\left(\tilde{\beta}^2(\frac{1}{\tau_\theta + \tau_\varepsilon + \tau_\delta} + \frac{\xi}{\tau_\delta}) + \left(\tilde{t}_i - \tilde{\alpha}S - \tilde{\beta}(\frac{\tau_\theta\bar{\theta} + \tau_\varepsilon S + \tau_\delta k_i}{\tau_\theta + \tau_\varepsilon + \tau_\delta} + \frac{(\tau_\theta + \tau_\varepsilon)k_i - \tau_\theta\bar{\theta} - \tau_\varepsilon S}{\tau_\theta + \tau_\varepsilon + \tau_\delta}\xi) - \tilde{\gamma}\right)^2\right)$$

To maximize $E(u_i|S,k_i)$, we get first order condition with respect to $\tilde{t}_i$

$$c_1\frac{\tau_\theta\bar{\theta} + \tau_\varepsilon S + \tau_\delta k_i}{\tau_\theta + \tau_\varepsilon + \tau_\delta} + c_2\left(\tilde{\alpha}S + \tilde{\beta}(\frac{\tau_\theta\bar{\theta} + \tau_\varepsilon S + \tau_\delta k_i}{\tau_\theta + \tau_\varepsilon + \tau_\delta} + \frac{(\tau_\theta + \tau_\varepsilon)k_i - \tau_\theta\bar{\theta} - \tau_\varepsilon S}{\tau_\theta + \tau_\varepsilon + \tau_\delta}\xi) + \tilde{\gamma}\right) = (c_1 + c_2)(\tilde{\alpha}S + \tilde{\beta}k_i + \tilde{\gamma}).$$

This equation satisfies with all $S$and k_i. Thus, we get

$$c_1\frac{\tau_\varepsilon}{\tau_\theta + \tau_\varepsilon + \tau_\delta} + c_2\tilde{\alpha} + c_2\frac{\tilde{\beta}(1-\xi)\tau_\varepsilon}{\tau_\theta + \tau_\varepsilon + \tau_\delta} = (c_1 + c_2)\tilde{\alpha},$$

$$c_1\frac{\tau_\delta}{\tau_\theta + \tau_\varepsilon + \tau_\delta} + c_2\tilde{\beta}(\frac{\tau_\delta}{\tau_\theta + \tau_\varepsilon + \tau_\delta} + \frac{(\tau_\theta + \tau_\varepsilon)\xi}{\tau_\theta + \tau_\varepsilon + \tau_\delta}) = (c_1 + c_2)\tilde{\beta},$$

and

$$c_1\frac{\tau_\theta\bar{\theta}}{\tau_\theta + \tau_\varepsilon + \tau_\delta} + c_2\left(\tilde{\beta}(\frac{\tau_\theta\bar{\theta}}{\tau_\theta + \tau_\varepsilon + \tau_\delta} - \frac{\tau_\theta\bar{\theta}\xi}{\tau_\theta + \tau_\varepsilon + \tau_\delta}) + \tilde{\gamma}\right) = (c_1 + c_2)\tilde{\gamma}.$$

Solve the equations, we get

$$\tilde{\alpha} = \frac{(c_1 + c_2)\tau_\varepsilon - c_2\tau_\varepsilon\xi}{(c_1 + c_2)(\tau_\theta + \tau_\varepsilon + \tau_\delta) - c_2(\tau_\delta + (\tau_\theta + \tau_\varepsilon)\xi)},$$

$$\tilde{\beta} = \frac{c_1\tau_\delta}{(c_1 + c_2)(\tau_\theta + \tau_\varepsilon + \tau_\delta) - c_2(\tau_\delta + (\tau_\theta + \tau_\varepsilon)\xi)},$$

$$\tilde{\gamma} = \frac{(c_1 + c_2)\tau_\theta\bar{\theta} - c_2\tau_\theta\bar{\theta}\xi}{(c_1 + c_2)(\tau_\theta + \tau_\varepsilon + \tau_\delta) - c_2(\tau_\delta + (\tau_\theta + \tau_\varepsilon)\xi)}. \qquad \square$$

The proof of first order condition in section 5.

$$Eu=-c_1[Var(\alpha S+\beta k_i+\gamma-\theta)+[(\alpha+\beta-1)\bar{\theta}+\gamma]^2]-c_2\beta^2Var(\delta_i)-\frac{1}{2}\lambda\tau_\delta^2$$

Let $f_1=-c_1[Var(\alpha S+\beta k_i+\gamma-\theta)+[(\alpha+\beta-1)\bar{\theta}+\gamma]^2]$

$$=-c_1\left[\alpha^2\left(\frac{1}{\tau_\theta}+\frac{1}{\tau_\varepsilon}\right)+\beta^2\left(\frac{1}{\tau_\theta}+\frac{1}{\tau_\delta}\right)+\frac{1}{\tau_\theta}+\frac{2\alpha\beta}{\tau_\theta}-\frac{2\alpha}{\tau_\theta}-\frac{2\beta}{\tau_\theta}+[(\alpha+\beta-1)\bar{\theta}+\gamma]^2\right],$$

$$f_2=-c_2\beta^2Var(\delta_i)-\frac{1}{2}\lambda\tau_\delta^2$$

$$=-c_2\frac{\beta^2}{\tau_\delta}-\frac{1}{2}\lambda\tau_\delta^2.$$

Take derivative with respect to $\tau_\delta$respectively.

$$\frac{\partial f_1}{\partial\tau_\delta}=-c_1\Bigg[-2\frac{\tau_\theta+\tau_\varepsilon}{\tau_\theta\tau_\varepsilon}\frac{c_1(c_1+c_2)^2\tau_\varepsilon^2}{[(c_1+c_2)(\tau_\theta+\tau_\varepsilon+\tau_\delta)-c_2\tau_\delta]^3}+2\frac{\tau_\theta+\tau_\delta}{\tau_\theta\tau_\delta}\frac{c_1^2(c_1+c_2)(\tau_\theta+\tau_\varepsilon)\tau_\delta}{[(c_1+c_2)(\tau_\theta+\tau_\varepsilon+\tau_\delta)-c_2\tau_\delta]^3}$$
$$-\frac{c_1^2}{[(c_1+c_2)(\tau_\theta+\tau_\varepsilon+\tau_\delta)-c_2\tau_\delta]^2}$$
$$+\frac{2}{\tau_\theta}\Bigg(\frac{-c_1^2(c_1+c_2)\tau_\varepsilon\tau_\delta}{[(c_1+c_2)(\tau_\theta+\tau_\varepsilon+\tau_\delta)-c_2\tau_\delta]^3}+\frac{c_1(c_1+c_2)^2\tau_\varepsilon(\tau_\theta+\tau_\varepsilon)}{[(c_1+c_2)(\tau_\theta+\tau_\varepsilon+\tau_\delta)-c_2\tau_\delta]^3}$$
$$+\frac{c_1(c_1+c_2)\tau_\varepsilon}{[(c_1+c_2)(\tau_\theta+\tau_\varepsilon+\tau_\delta)-c_2\tau_\delta]^2}-\frac{c_1(c_1+c_2)(\tau_\theta+\tau_\varepsilon)}{[(c_1+c_2)(\tau_\theta+\tau_\varepsilon+\tau_\delta)-c_2\tau_\delta]^2}\Bigg)\Bigg]$$

$$=-c_1\Bigg[2\frac{c_1(c_1+c_2)[-(c_1+c_2)(\tau_\theta+\tau_\varepsilon)\tau_\varepsilon+c_1(\tau_\theta+\tau_\delta)(\tau_\theta+\tau_\varepsilon)-c_1\tau_\varepsilon\tau_\delta+(c_1+c_2)\tau_\varepsilon(\tau_\theta+\tau_\varepsilon)}{\tau_\theta[(c_1+c_2)(\tau_\theta+\tau_\varepsilon+\tau_\delta)-c_2\tau_\delta]^3}$$
$$+\frac{c_1[-c_1\tau_\theta+2(c_1+c_2)\tau_\varepsilon-2(c_1+c_2)(\tau_\theta+\tau_\varepsilon)]}{\tau_\theta[(c_1+c_2)(\tau_\theta+\tau_\varepsilon+\tau_\delta)-c_2\tau_\delta]^2}\Bigg]$$

$$=-c_1\left[\frac{2c_1^2(c_1+c_2)(\tau_\theta+\tau_\varepsilon+\tau_\delta)\tau_\theta}{\tau_\theta[(c_1+c_2)(\tau_\theta+\tau_\varepsilon+\tau_\delta)-c_2\tau_\delta]^3}+\frac{c_1[-c_1\tau_\theta-2(c_1+c_2)\tau_\theta]}{\tau_\theta[(c_1+c_2)(\tau_\theta+\tau_\varepsilon+\tau_\delta)-c_2\tau_\delta]^2}\right]$$

$$=-c_1\left[\frac{2c_1^2(c_1+c_2)(\tau_\theta+\tau_\varepsilon+\tau_\delta)}{[(c_1+c_2)(\tau_\theta+\tau_\varepsilon+\tau_\delta)-c_2\tau_\delta]^3}-\frac{c_1^2+2c_1(c_1+c_2)}{[(c_1+c_2)(\tau_\theta+\tau_\varepsilon+\tau_\delta)-c_2\tau_\delta]^2}\right]$$

$$\frac{\partial f_2}{\partial\tau_\delta}=-2\frac{c_1^2c_2(c_1+c_2)(\tau_\theta+\tau_\varepsilon)}{[(c_1+c_2)(\tau_\theta+\tau_\varepsilon+\tau_\delta)-c_2\tau_\delta]^3}+\frac{c_1^2c_2}{[(c_1+c_2)(\tau_\theta+\tau_\varepsilon+\tau_\delta)-c_2\tau_\delta]^2}-\lambda\tau_\delta$$

Sum $\frac{\partial f_1}{\partial\tau_\delta}$and $\frac{\partial f_2}{\partial\tau_\delta}$, we have

$$\frac{\partial f_1}{\partial\tau_\delta}+\frac{\partial f_2}{\partial\tau_\delta}$$

$$=\frac{-2c_1^3(c_1+c_2)(\tau_\theta+\tau_\varepsilon+\tau_\delta)-2c_1^2c_2(c_1+c_2)(\tau_\theta+\tau_\varepsilon)}{[(c_1+c_2)(\tau_\theta+\tau_\varepsilon+\tau_\delta)-c_2\tau_\delta]^3}+\frac{c_1^2c_2+c_1^3+2c_1^2(c_1+c_2)}{[(c_1+c_2)(\tau_\theta+\tau_\varepsilon+\tau_\delta)-c_2\tau_\delta]^2}-\lambda\tau_\delta$$

$$=\frac{-2c_1^2(c_1+c_2)^2(\tau_\theta+\tau_\varepsilon)-2c_1^3(c_1+c_2)\tau_\delta}{[(c_1+c_2)(\tau_\theta+\tau_\varepsilon+\tau_\delta)-c_2\tau_\delta]^3}+\frac{3c_1^2(c_1+c_2)}{[(c_1+c_2)(\tau_\theta+\tau_\varepsilon+\tau_\delta)-c_2\tau_\delta]^2}-\lambda\tau_\delta$$

$$=\frac{-2c_1^2(c_1+c_2)^2(\tau_\theta+\tau_\varepsilon)-2c_1^3(c_1+c_2)\tau_\delta+3c_1^2(c_1+c_2)[(c_1+c_2)(\tau_\theta+\tau_\varepsilon+\tau_\delta)-c_2\tau_\delta]}{[(c_1+c_2)(\tau_\theta+\tau_\varepsilon+\tau_\delta)-c_2\tau_\delta]^3}\quad-\lambda\tau_\delta$$

$$= \frac{c_1^2(c_1+c_2)^2(\tau_\theta+\tau_\varepsilon)+c_1^3(c_1+c_2)\tau_\delta}{[(c_1+c_2)(\tau_\theta+\tau_\varepsilon+\tau_\delta)-c_2\tau_\delta]^3}-\lambda\tau_\delta$$

$$= \frac{c_1^2(c_1+c_2)[(c_1+c_2)(\tau_\theta+\tau_\varepsilon+\tau_\delta)-c_2\tau_\delta]}{[(c_1+c_2)(\tau_\theta+\tau_\varepsilon+\tau_\delta)-c_2\tau_\delta]^3}-\lambda\tau_\delta$$

$$= \frac{c_1^2(c_1+c_2)}{[(c_1+c_2)(\tau_\theta+\tau_\varepsilon+\tau_\delta)-c_2\tau_\delta]^2}-\lambda\tau_\delta=0.$$

We get $c_1^2(c_1+c_2)=\lambda\tau_\delta[(c_1+c_2)(\tau_\theta+\tau_\varepsilon+\tau_\delta)-c_2\tau_\delta]^2$ or $c_1+c_2=\lambda\tau_\delta\left[\tau_\theta+\tau_\varepsilon+\tau_\delta-\frac{c_2}{c_1}(\tau_\theta+\tau_\varepsilon)\right]^2$ □

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